Part III

Conclusions
Chapter 7

Modeling

7.1 Introduction

For an engineer it is not sufficient to acquire just enough knowledge to get a “correct” answers when solving problems – in some senses this will limit learning and application of technical knowledge. It is important that students acquire knowledge that make them able to confront new situations, to develop skills giving solutions or increasing that knowledge with new approaches. But this is a large process; that we consider as steps where the student has to develop a special level of thinking and to integrate elements to be able to manage complex concepts to solve specific problems.

According to Sánchez (1999):

One of the fundamental problems that the mathematics education in the university raise is the high level of abstraction in the content of the subjects and are taught in the scientific technique carrer. Historically Núñez and Font (1995), regarding this problem, has taken two opposite directions. The first is to present the concepts in the most general way possible, explaining the contents as a deductive structure from some axioms or general principles. It is sometimes called “euclideanisation” of the mathematics education, making reference to an axiomatic procedure of the presentation of the Euclidean elements that has had a strong influence in history of mathematics (Hernández 1995).

The second direction consists in working with concepts in different and particular contexts, leading to an increased the functionality and significance. This kind of approximation of the problem is that, according to our criteria, facilitates the learning, over all, in the contexts where superior mathematics is studied as a tool of work. For example, in the studies of engineering, where the adequate contextualization of the contents in the subjects of mathematics can increase the motivation of the students and improve the learning.

7.2 Modeling and Solving Problems

The term “modeling” is new and can be defined as a process of intentional construction that through a system symbols, a perception of the experience of reality perceived
by the one who is modeling. Sometimes a model is defined as an expression in the mathematical language, but it is right to say that to this restrictive form of conceiving the process of modeling a pragmatic or heuristic modeling can also be added.

The models are built to give sense to the data that we use, in this way, the models are an inevitable element in the construction of the reality and, at the same time, they give us interpretative tools to reconstruct it. Without models it is not possible to generalize. It is important to understand how these artefacts are built, through them we make sense of the things we are acting on.

The mathematical modeling consists in formulating a real technical problem in mathematical way, solving and interpreting the results in terms of the problem and of the situation studied. Some of the intentions that they construct a model are:

1. To obtain answers on what can happen which are difficult to realize in a real world test
2. To facilitate the comprehension of the technical phenomena
3. To offer a more useful vision of the mathematics

The views expressed by Fernández (no date) affirm that some of the intentions by which a model is constructed are:

1. Obtaining answers on what can happen in front of a physical phenomenon
2. To influence the experimentation or the observations that can take place later
3. To facilitate and to promote the progress and comprehension of the technical phenomena
4. Models promote the mathematics and the art to build models

According Niss (1989) modeling is the art to apply mathematics to situations of real life. When Niss talk about “the art”, he is pointing out, as relevant, the group of abilities to construct or discover models. And this definition has implicit the heuristic character in the process of modeling.

Bloor (1998) says that when we learn mathematics, we resort to our group of experiences about the behavior of material objects. The organization of physical objects supply models to our mental processes, for example the processes of mathematical reasoning are shadows of the physical operations with objects. But there exists a large group of research that criticizes the idea that the numbers are proprieties of the external things. They affirm that the number that we assign to things depend on how we have previously classified those things, and this depends on our purposes. The idea of a number as something subjective is also criticized. On the other hand there exists the formalist program where there are no mathematical objects, just ostentatious symbols. For the extreme formalism, the thing that exist is rules to deduce formulas from other formulas. But the formulas do not refer to anything. Formulas are just a string of symbols that does not have any meaning. Neither do they have any real value assigned (symbols without meaning) as the results in Study 2 and 3 (see section 3.3 and 5.3).
7.2.1 Example of mathematic model in electrical engineering

This example was made by Dr. Robert W. Grubbström. In a interview made to Dr. Grubbström (Study 4), he explains how mathematics can be interpreted in a physical phenomena.

Mathematics can be interpreted. Mathematics is symbols concerning quantities: real and imaginary numbers.

But then you attach some kind of reality or modeling when using in mechanics or electrical engineering.

---

1Expert in the Laplace transform topic and he has a large number of publications in that matter.
Then you say: “this is current” or “this is voltage” or “this is something else”. And you show that inductivity has a certain effect on the equation.

And it’s easy to model the equations and transform them.

Then you are interpreting the entities and the formulas, and you can do it in physics or in electrical engineering.

In economic theory, you look at cash flow, that becomes present value, in interpreting, you can put it in many places.

According with the analysis made in the Study 4 we found that the “model” is a link or way to connect the abstract thinking with the material aspect in engineering education as shown in Figure 7.1. And in the model world mathematic, physic and technology are interacting as shown in Figure 7.2.

Figure 7.1 represent a model is a tool to solve problems and real situations, by a process that can be summarized in three steps:

**Step 1**: In the material and real world, we express a physical situation, interest, problem or uncertainty with a model.

**Step 2**: We move to abstract world developing the system, sometimes it is necessary to do it by other models, for instance mathematical methods.

**Step 3**: We get back to the physical world, interpreting the model (mathematical results) in physical terms.
Figure 7.1: A model is seen as a link between the “material” and the “abstract” aspect

Figure 7.2: Model of triangulation of mathematics, physics and technological aspect
7.2.2 Example of dividing the land

All mentioned before can be illustrated with a simple example:

We have a real problem for to solve: “A man wants to divide and share a land of 2ha among his three children, in the way that to the second child gets 35% more land than the first and the third child 10% more than the second child”.

**Step 1:** We establish a model for this real situation (Material aspect):

We assign an algebraic symbol to each child of the man:

- Part of the land corresponding to first child $\Rightarrow a$
- Part of the land corresponding to second child $\Rightarrow b$
- Part of the land corresponding to third child $\Rightarrow c$

**Condition** Mathematical model based in real conditions:
Mathematical world  Real Physical world

\[ a + b + c = 20000 \text{m}^2 \quad (7.1) \]

Total area of 2ha

\[ b = a + 0.10a \quad (7.2) \quad \text{10\% more land than the first child corresponds to the second child} \]

\[ c = b + 0.35b \quad (7.3) \quad \text{10\% more land than the second child corresponds to the third child} \]

Step 2: To develop the system (abstract aspect).

Substituting in \((7.1)\) the equivalence of \((7.2)\) and \((7.3)\) where:

From \((7.2)\):
\[
 b = a + 0.10a = 1a + 0.10a = 1.10a
\]

(7.4)

From \((7.3)\):
\[
 c = b + 0.35b = 1b + 0.35b = 1.35b
\]

(7.5)

subsisting \((7.4)\) in \((7.3)\):
\[
 c = 1.35(1.10a) = 1.4850a
\]

(7.6)

Substituting \((7.4)\) and \((7.6)\) in \((7.1)\):
\[
 a + 1.10a + 1.4850a = 20000
\]

3.585a = 20,000
\[
 a = 20000/3.585 = 5578.8006 \text{m}^2
\]

(7.9)

Then:
\[
 b = 1.10a = 6136.6806 \text{m}^2
\]

(7.10)
\[
 c = 1.4850a = 8284.5188 \text{m}^2
\]

(7.11)

Step 3: To apply the results to the original situation (material aspect).

The man has to share the land among his three children in the following way:

From \((7.9)\): \( a = 5578.8006 \text{m}^2 \) corresponds to the first child

From \((7.10)\): \( b = 6136.6806 \text{m}^2 \) corresponds to the second child

From \((7.11)\): \( c = 8284.5188 \text{m}^2 \) corresponds to the third child
**Checking**  \( The \ addition \ of \ the \ quantities \ of \ each \ children \ has \ to \ be \ 20000m^2. \)

\[
5578.8006m^2 + 6136.6806m^2 + 8284.5188m^2 = 20000m^2 \\
\text{(7.12)}
\]

This example shows how the models help us to make a representation of physical situations and to solve them in the abstraction. The material world correspond to the man with the situation of sharing the land among his children. The models helped us to represent the situation and to establish a system to solve it in the abstract aspect (in this case by mathematics) and there we got a result that was possible to interpret in the material world and solve our original problem. As shown in Figure 7.1.
7.3 Modeling and the Transforms

More complex example an also illustrate the “Material-Abstract Model”. And it is the case of Transforms.

Through Transforms we move between different domains. In the real material world (real world) we work in the time domain, and it means that the time is one of the principal independent variable, and in general, it is unique. Then, we transform the system, and in consequence we change the domain and another independent variable appear instead of the time. In general this variable is the frequency. When we find the solution in the frequency domain, we can apply that solution to the reality, but we have to come back to the time domain; this inverse process is called inverse transform. The way to work in the frequency domain will depend of the “Transform” that has been used. Mathematicians and Physicist have developed some mathematical transforms that make the solution of problems easier. Among transforms, four are considered the most relevant: the Phasorial transform, the Laplace transform, the Fourier transform, and the Z transform. Each of them have their own particularities with their specific fields of application, but all of them focus in facilitating the process in different areas, for instance, the electricity. When we come back to the time domain, we can use our engineering knowledges and skills to apply the solutions found to real cases or situations. This processes are the daily life of a engineer. This process is, for example, similar to the process, to loosen a nut. If you use a plier, it is possible that you will remove the nut but you are likely to damage both the screw and the nut. But if you use the appropriate spanner, it will be more easy to loosen the nut. There are many kinds of spanners, some better than others, depending on what kind of screw and the nut. At the same time the Transforms are used, depending on their efficient on the case to solve.

Figure 7.1 represents that it is possible to use the Model as a tool to solve situations of the material world problems by abstraction. As Platon said, beyond the physical things, there exists another kind of things called “ideas”. For example, the idea of triangle, there also exist figures drawn on paper that correspond to this idea. The ideas are superior to the physical objects. Because the ideas show the properties of the physical objects in perfect way and because of this we know more about those physical objects by looking at their ideas than the objects in their selves.

According to Hickman (1990) “the tool becomes a part of the active productive skill brought to bear on the situation. The purpose of the tool is to reorganize the experience in some way that will overcome its disparity, its incompatibility, or its inconsistency. A tool is in this sense a theory, a proposal, a recommended method or course of action”.

Piaget (1979) considers that abstraction is the ability that let us construct concepts, but he doesn’t consider that this construction is just the result of the comparison. He believes that our actions are very important to abstract concepts. Piaget distinguishes the simple or empiric abstraction from the reflexive or logic-mathematical abstraction. Those kind of abstractions works in coordinate way in most of general situations. This way of abstraction explain the construction of mathematical objects.

Some research affirm that there exists special necessities in the education in university level, specially in engineering education:

“In our opinion, one of the problems of the education of the mathematics and the physics in the first courses of the Spanish universities, specially in
Figure 7.3: Actions
schools of engineering, where students, in many occasions do not understand the need of a scientific solid formation. Does this owe to a great extent to the pragmatic domineering ideology with regard to their own formation ‘this why?’ that often meets reinforced by the opinions of a wide sector of the professorship.” (Sanchez 1999)

In an interview in Study 4 (section 6) a teacher (I5) expressed: All the mathematical knowledge is there, and also by reading a book it is possible to acquire knowledge. But the problem is to link that knowledge with the real event.

### 7.3.1 Example of Proofing Motors

Modelling is important in engineering and the relevance is stated in the following interview (I5) from Study 4:

> “An application of the Laplace transform is by Matlab, simulating the Bode diagram: controllability and stability of the system. In fact it is one important part where the simulation area is working more right now, for example talking about simulation of big and small motors, perhaps rise the question: why is the reason to do it?, when you take it in the practical part, in the real life?”

But we can remember what happen in the past, producing automobiles?

In the past, to make proofs it was necessary to crash cars in the wall to observe:

1. The resistance of the material.
2. Level of protection to the driver
3. How much damage the car suffers every crash

Then, the question is raised:

“How much money is spent in every probe?”

So, it is better if I crash one car obtaining parameters of it and these parameters can be used in simulations and I do not have to crash cars against the wall every time.
Through the simulation I have my parameters and I can obtain my proofs.
Similarly with electric motors:
It is not necessary to test the rotor and stator of a machine every time which would require cutting and proofing the motor. Sometimes it is possible to work with small motors and translate (scale) results to larger motors. If somebody wanted to test the failure rate of a motor one would have to break the axle or to force the machine axle to observe kind of failures and fractures. Then one would as a test break the motor to get further results, but these results are valid to make an analysis of a larger motor.
Chapter 8

The Techné Pyramid

8.1 Background

Piaget (1979) considers that learning is constructive. He says that to understand is a construction made by yourself. In the sense, that we can help students to acquire mathematical concepts by didactic materials, questions and teachers’ explanations, but only by their own effort they can really learn. According John Dewey (1938) we affirm that students are the protagonist and they have to be the actor of their self learning. No one can learn by them.

Kolb (1984) propose the modeling as a necessary element in the process of learning. Kolb define the learning as the process to create knowledge through the transformation of the experience.

Generally it is assumed that the university students have the intellectual and methodological maturity necessary to understand and to integrate the new concepts, dedicating small or no attention to the contextualization of the advanced knowledge. Nevertheless, nowadays a great part of the investigations in didactics of science or engineering admits that the discontextualization is one of the central problems in the comprehension of the mathematical concepts and in general of the scientific theories (Font 2000).

By other hand, as pointed by Thompson (1992) other views exist: Teachers holding constructivist view of mathematics are expected to adopt teacher-student interaction mode of instruction by allowing students to explore and investigate while teachers reside in their classrooms as facilitators. Problem solving is central to teaching for constructivist mathematics teachers where purposeful activity stems from problem situations that require reasoning and creative thinking, gathering and applying information, discovering, inventing, communicating and testing ideas. Consequently, the classroom takes on a constructivist environment.

8.1.1 Techné versus Epistémé

From Stanford Encyclopedia of Philosophy (Parry, 2003):

Epistémé is the Greek word most often translated as knowledge, while techné is translated as either craft or art /.../ Outside of modern science, there is sometimes skepticism about the relevance of theory to practice because it is taught that theory is conducted at so great a remove from
The Technê Pyramid

reality, the province of practice that it can lose touch with it. /.../ The relation then, between epistêmê and technê in ancient philosophy offers an interesting contrast with our own notions about theory (pure knowledge) and (experience-based) practice. There is an intimate positive relationship between epistêmê and technê, as well as a fundamental contrast. /.../

1. Xenophon

Xenophon’s only sustained discussion of epistêmê and technê are in two of his Socratic works, Memorabilia and Oeconomicus. The Memorabilia recounts conversations which Socrates held on a variety of topics; the Oeconomicus is a conversation largely devoted to one, i.e., the art of running a successful estate and household. In these works, knowledge is intimately tied to knowing how to do things, especially the more organized kind of knowing-how designated by technê. /.../

2. Plato

In Plato’s dialogues the relation between knowledge (epistêmê) and craft or skill (technê) is complex and surprising. /.../ Knowledge (epistêmê) is the ability to know the real as it is.” /.../

3. Aristotle

He begins with a distinction between two parts of the rational soul, calculating part (to logistikon) and the scientific part (to epistêmonikon). /.../ The full account of epistêmê in the strict sense is found in Posterior Analytics, where Aristotle says that we think we know something without qualification (epistasthai...haplôs) when we think we know (gignôskein) the cause by which the thing is, that is the cause of the thing, and that this cannot be otherwise. /.../

4. The Stoics

Among the Stoics, the relation between epistêmê and technê is the richest and most focused of all the accounts we have so far considered. That relation is enmeshed in the Stoic account of virtue, in which the two notions of knowledge and craft flow together in forming the science and art of living. /.../

5. Alexander of Aphrodisias

Alexander introduces the idea of stochastic technê – and idea important for the Stoic explanation of virtue that we have just seen. /.../ Alexander calls the technai that use this kind of syllogism stochastic. /.../

6. Plotinus

As might be expected, Plotinus’ philosophy does not have much use for the concept of technê. Its account of knowledge is fuller than that of craft and is close to Aristotle’s idea of epistêmê in the strong sense.

According to Mitcham (1994):

“In philosophical works, techne comes to be conceived not only as an activity of some particular sort or character, but as a kind of knowledge of
Technology includes more than material objects such as tools and machines and mental knowledge or cognition of the kind found in the engineering sciences.

According Mitcham (1994) “[t]echnology is not so much the application of knowledge as a form of knowledge ...”.

In the words of Layton (1976) this is stated as:

“Engineering science often differs from basic science in important particulars. /.../ Engineering theory and experiment came to differ from those of physics because it was concerned with man-made devices rather than directly with nature. /.../ By its very nature, therefore, engineering science is less abstracted and idealized; it is much closer to the ‘real’ world of engineering. Thus, engineering science often differs from basic science in both style and substance. Generalizations about ‘science’ based on one will not necessarily apply to the other.”

This difference is one of our arguments behind that we have developed a model somewhat different from the SOLO-taxonomy (Biggs and Collis, 1982) and the taxonomy of Bloom (Bloom et al., 1956; Anderson and Krathwohl, 2001).

To emphasize that our model is developed with the education of engineers in mind, with the forms of knowledge important for an engineer in mind, we have named our model Techne Pyramid.

Techne were used by the Greeks at the time of Plato and Aristotle to designate a productive skill (see for example Parry, 2003; Hickman, 1990; Mitcham, 1994; Verbeek, 2005). Techne involved both knowledge and ability directed toward fulfillment of some aim. It involved a rational professional skill beyond simple experience. For most Greeks at his time the concept of episteme, theoretical knowledge, dealt with the immutable, with the knowledge what were thought of as unchangeable.

Thus techne stood in the middle between mere experience, empeiria, and theoretical knowledge, episteme. Thus it occupied a space similar to that of mediating tools (artifacts) in theory of mediated learning (see section 9.5) there tools/artifacts were simultaneously seen as having material and ideal (conceptual) nature or to my idea of modeling as a link between the material and abstract.

According to Hickman (1990):

“Technē was for the Greeks a pro-duction, a leading toward, and a construction, a drawing together, of various parts and pieces in order to make something novel. Technē was thus central to the thought of the Greeks in the sense of having been an important element in their form of life. But it was also centrally located for them between state of nature and finished artifact, between necessity and chance, and between theoretical certainty and unstructured experience”.

Technē is thus very much related to the design level in our model of Techne Pyramid
8.1.2 Bloom’s taxonomy

Benjamin Bloom and co-workers (Bloom et al., 1956) created his well-known taxonomy originally for categorizing level of abstraction of questions that commonly occur in educational settings, specially those on exams.

The categories in the original version of Bloom’s taxonomy are:

1. Knowledge
2. Comprehension
3. Application
4. Analysis
5. Synthesis
6. Evaluation

L Anderson and co-workers (Anderson and Krathwohl, 2001) have revised Bloom’s original taxonomy have proposed the following taxonomy for the cognitive process:

1. Remember – Retrieve relevant knowledge from long term memory
2. Understand – Construct meaning from instructional messages, including oral, written, and graphic communication
3. Apply – Carry out or use a procedure in a given situation
4. Analyze – Break material into its constitutive parts and determine how the parts relate to one another and to an overall structure or purpose
5. Evaluate – Make judgments based on criteria and standards
6. Create – Put elements together to form a coherent of functional whole; reorganize elements into a new pattern or structure

8.1.3 SOLO

Another widely used taxonomy is the SOLO-taxonomy (Structure of Observed Learning Outcome) developed by Biggs and Collis (1982):

1. Pre-structural: here students are simply acquiring bits of unconnected information, which have no organization and make no sense.
2. Unistructural: simple and obvious connections are made, but their significance is not grasped.
3. Multistructural: a number of connections may be made, but the meta-connections between them are missed, as is their significance for the whole.
4. Relational: the student is now able to appreciate the significance of the parts in relation to the whole.
5. Extended abstract level: the student is making connections not only within the given subject area, but also beyond it, able to generalise and transfer the principles and ideas underlying the specific instance.
8.2 What is Techné Pyramid?

Technology became something accessible for the people, but that does not imply which kind of knowledge is within the technology. For example, the computer, being able to use the computer does not imply that we have a deeper knowledge of what the computer is. We affirm that there is no absolute “scientific method” for getting scientific knowledge. The students can learn and repeat but that does not mean knowledge. For technical competence “know” and “how” is important, and the Techné Pyramid is a model to propose how to reach this, considering that every case is particular and there is no strict way to follow, but it is applicable to determine specific agreements and/or objectives, discarding not necessary elements. Considering that technology makes science as well as science making technology. Technology change societies, social structures make technologies. Technologies have their own language and it is not necessarily easy to understand.

We propose a model that has some similarities to both the SOLO and the Bloom’s taxonomy. We illustrate the model in Figure 8.2 with a ‘skill’ and ‘knowledge’ pyramid called “Techné Pyramid” with three levels whose base is a group of knowledges, skills and experiences going through an ‘analysis’ level to reach the ‘design’ level on the top of the pyramid.

According to Østerberg Rump (2005) in the context of engineering knowledge proposes that engineering starts with problems, including designs problems, and the engineer needs solid knowledge for problem solving:

- Scientific
- Empirical
- Experimental

We propose the following model, considering the points above in the engineering context.
The Techné Pyramid

First level: “Basic”
Second level: “Analysis”
Third level: “Design”

Figure 8.2: Techné Pyramid
This model is applicable in context of solving problems, where we have the solution of a specific problem as the objective, and we describe the process of reaching that objective. We consider reaching this objective a constructive process building the necessary elements required to solve the problem. In this building process we consider several steps, or levels of skills, knowledges, experiences and other elements, where each level is supported by the previous level, and is in turn supporting the next. Each new level of elements becomes more complex, and requires that the elements from the previous level interact, and integrate. In this way the pyramid represents a large number of disconnected elements in the lower levels, and as we get closer to the top of the pyramid, the number of elements becomes fewer, more complex and more specific to the problem (Figure 8.3).

Here it is important to sign out that this model of pyramid cannot be considered as isolate. In the sense that when it is reached the solution of the problem and was produced something new, this new element applied in other context, with other specific situations becomes a member of the group of elements that support another objective (Figure 8.4).

We can exemplify this situation with the case of the Laplace transform in engineering education. The Laplace transform is considered a difficult concept, not easy to understand without previous mathematical knowledge. Carstensen and Bernhard (2004) have studied how engineering students solve electric circuits using the Laplace transform in labwork. And according with the Study 4 (chapter 6) the Laplace transform concept is not on a basic level of understanding, the students have to use other resources of thinking.
In Figure 8.4 we describe the different levels of the Techné Pyramid. In the first level the memory has the most important role and the basic concepts are seen as independent ‘islands’ between which the students do not see connections. In the second level the student see the complexity and are able to make an analysis and for example make connections between concepts for a specific purpose. In the third level knowledge and skills from the previous levels are used to produce and design something new.

We will relate the levels of our proposed pyramid with commentaries in interviews with engineering educators regarding the importance and applicability of the Laplace transform.

8.2.1 First level is called BASE

Considering that for to solve problems is necessary solid theoretical base, the contents of this level is like different “islands” of concepts, skills, knowledges and more aspects, that correspond to those elements that, are not necessarily managed in deeper way but in any way those elements are part of the background of the engineering student. In this level the memory has big place. These “islands” of elements are not necessarily linked to each other. In Figure 8.5 these “islands” are illustrated from a transient response lab (Bernhard and Carstensen, 2004).

To illustrate this level, we can propose the following example. In the subject of Electricity it is quite common to manage different concepts as current, voltage, potential difference, etc. and to give specific results and also it is possible to do it without linking the concepts involved, since a deep understanding of these concepts are not
8.2 What is Techné Pyramid?

To produce something new
Analysis, abstract thinking.
relation and/or interaction between different elements.

Previous knowledge, complexity and new experiences interacting to solve a problem.

Groups of knowledges, experiences, skills, etc., not necessary linked, either managed in deeper way but are the base and support for other elements.

Figure 8.5: Description of Techné Pyramid
necessary this subject. But when these concepts form the base for other, so called, “complex concepts” a deeper understanding is necessary.

In one of the interviews (Study 4) a teacher expressed:

“The Laplace transform is not difficult to understand, the problem is that students doesn’t have enough basic mathematical background or they don’t know the importance and benefits of this topic.” (I15).

8.2.2 Second level is called ANALYSIS

In this part the abstract thinking has the big place. The students develops links between different “islands”. Previous concepts help to understand a new complex concept. See Figure 8.6 as an illustration.

(a): Benny’s lived object of learning in this first part of the lab  
(b): Tess’ lived object of learning in this first part of the lab  
(c): Links made at the end of the lab-work in the new course

**Figure 8.6:** Example of analytical concepts and the connections made between them in a transient response lab (Carstensen and Bernhard, 2004) there the concepts correspond to the first level in our pyramid and the connections to the second level

In the following explanations from interviews (Study 4) we can see that all teachers are talking about a more complex level with the use of the Laplace transform, where it is necessary to include and to link other concepts. And also “the model” to work and help to understand is included.
**Table 8.1: Comments from teachers**

<table>
<thead>
<tr>
<th>Interview</th>
<th>Teacher</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>I4</td>
<td>...Almost always not the simple control but the second step in the control ‘manufacturing process’. Because the simple processes then you don’t care about the system what it is? You don’t do a model of it and then you don’t have to use the Laplace transform, you can do more about logical reasoning and you don’t have to do the model of the system but then in the next step when you try to analyse in some way you have to use the LT in automatic control to control this application so in that case the subject of automatic control is application driven.</td>
<td></td>
</tr>
<tr>
<td>I10</td>
<td>The frequency answer from a system described by a differential equation (resonance, static gain, attenuations, etc.) let the advanced analysis of the proprieties and behaviour of a physic system then, the design of controllers, regulators and filters.</td>
<td></td>
</tr>
<tr>
<td>I1</td>
<td>I thing the LT is such concept that train the abstract thinking so It is not much the matter how to use for is more the matter how could I train my thinking... The Laplace transform is a complex concept, is talking about an especial level of thinking.</td>
<td></td>
</tr>
<tr>
<td>I4</td>
<td>The real knowledge is when you can really relate those transforms and see which one is useful in which context. The teacher is talking about the importance of links between elements, in this case the transforms with specific context.</td>
<td></td>
</tr>
<tr>
<td>I9</td>
<td>I believe that theory should be taught before application... But the theory takes long time to mature and therefore the sooner you start learning theory and abstract thinking the better because it takes time before you actually grasp it. I remember when I went to high school I didn’t have good marks in mathematics the first term, I though it was very difficult, but later on I improved so I had the highest grade when I left, but still I had difficulties because it took time to mature. So I think you should start early, maybe you don’t have to have much but don’t avoid theory in the beginning in university education. I think that’s dangerous. To start early with mathematics help you to let the knowledge mature, to reach the analysis level, otherwise you will stay just in the basic level and becomes difficult to manage more complex situations.</td>
<td></td>
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</tbody>
</table>
8.2.3 Third level is called DESIGN

When something “new” is produced as a result of the previous steps worked or focused in solving a specific necessity.

“Engineering design is a systematic effort to save effort. But what, more precisely, is this saving of effort? /.../ The term ‘efficiency’ has its roots in the Latin efficere (to produce, effect, or make). /.../ In English, ‘efficiency’ traditionally meant the operative agency or power of something or someone to get something done, to produce results.” Mitcham (1994)

Some results of the research some students consider the Laplace transform a topic that consist just to apply mathematical formulas without more transcendence but according with the experience of a teacher we found:

<table>
<thead>
<tr>
<th>Interview Teacher</th>
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<tbody>
<tr>
<td>I7</td>
</tr>
<tr>
<td>If the objective were to solve differential equations, I would agree with the student that argued that the Laplace transform is just mathematical formulas, but in the subject of Electric Circuit Theory is fundamental to make the student notice the utility of the Laplace transform solving electric circuits, but over all to interpret the electric circuit behaviour and to give bases to design, for example the case of Filers.</td>
</tr>
</tbody>
</table>

Table 8.2: Comments from teachers

The process of learning in engineering education focus on solving specific problems concluding, some times in the design. All the groups of concepts, that engineering students have to learn, have a purpose. But this aspect is not always clear to the student.

Schön (1987) expresses:

A designlike practice is learnable but is not teachable by classroom methods. And when students are helped to learn to design, most useful to them are more like coaching than teaching-as in reflective practicum.

Several features make this process learnable, coachable, but not teachable.

1. Skillful designing is a kind of knowing-in-action. It is possible to describe rules used in designing. But some of the most important rules cannot be followed in a simple, mechanical way.

2. Designing is a holistic skill. One cannot learn it in a molecular way, by learning first to carry out smaller units of activity and then to string those units together in a whole design process; for the pieces tend to interact with one another and to derive their meaning and characters from the whole process in which they are embedded. Its true, of course, that design processes may be broken into component parts by strategies of decomposition useful both to practice and coaching. Although a larger design problem can be broken into parts, the total
solution is not a sum of the smaller ones. When a student has learned to carry out smaller units of design activity but has not yet learned how to integrate them into a larger design process, the nature off the larger whole is likely to seem confusing.

3. Skillful designing depends on a designer’s ability to recognize and appreciate desirable or undesirable design qualities. A student may be helped to recognize and appreciate quality like “enclosure” or “directionality”, however, without recourse to verbal description.

4. The description of one’s own knowing-in-action is itself a skill, and designers may posses it in greater or lesser degree. Designers can learn to make better descriptions of designing-more complete, accurate, and useful for action-by continued reflection on their own skillful performance. How far they can go in this direction should remain an open question, however, testable in each new effort at description.

5. Designing is a creative activity. For several reasons, then, a design-like practice cannot be conveyed to students wholly or mainly by classroom teaching:
   - The gap between a description of designing and knowing-in-action that corresponds to it must be filled by reflection-in-action.
   - Designing must be grasped as a whole, by experiencing it in action.
   - Designing depends on recognition of designing qualities, which must be learned by doing.
   - Descriptions of designing are likely to be perceived initially as confusion, vague, ambiguous, or incomplete; their clarification depends on a dialogue in which understandings and misunderstandings are revealed through action.
   - Because designing is a creative process in which a designer comes to see and do things in new ways, no prior description of it can take the place of learning by doing.

From all this, of course, it doesn’t follow that students cannot learn to become proficient at designing in all the senses listed above. They can do so, and they can be helped by exposure to explicit descriptions of designing.

From Study 4 we had some comments that support this part of the model:
If the objective were to make the student able to solve differential equations, then I would agree with the student (when student says that it is not necessary to learn the Laplace transform, are just mathematical operations but the subject of Electric Circuits Theory is necessary to make students see the utility of the Laplace Transform when they solve an electric circuit and over all to interpret the behavior of it for a general case without solve it and to give the bases for the design.

It depends on his job or her job. I think that it’s basic use the transform in circuit theory, very basic method, and you need to, if you are an electrical engineer, you need transforms to analyze frequency better. You need to be able to solve circuit problems, looking frequency, understand questions of frequency and stability which are easily analyzed in transforms compared to time. Looking at the time domain, sometimes it’s very messy, I think definitely necessary for a university qualified engineer; I think that technicians don’t use it so much maybe. They might suffice that they just use it superficially as methodology and they don’t bother too much, maybe is behind.

One difficulty to understand the Laplace transform for the students is that they don’t see the application in the object world, and then they don’t understand what they are doing. They have to use more the abstract thinking to work with changes of spaces (time and frequency domain).

Many times, the student doesn’t have the enough experience about the “problems of the world” to know how and where he can apply his knowledge and for this reason he cannot see the importance among different topics.

<table>
<thead>
<tr>
<th>Interview</th>
<th>Teacher</th>
<th>Comment</th>
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<tbody>
<tr>
<td>I12</td>
<td></td>
<td>He explains briefly the three levels in Electric Circuits: first to solve differential equations, then the second to know the utility of the Laplace transform to interpret the behavior of the electric circuit and to give the bases for the next third step that is design. The Laplace transform, here, is located in the level of analysis.</td>
</tr>
<tr>
<td>I9</td>
<td></td>
<td>It depends on his job or her job. I think that it’s basic use the transform in circuit theory, very basic method, and you need to, if you are an electrical engineer, you need transforms to analyze frequency better. You need to be able to solve circuit problems, looking frequency, understand questions of frequency and stability which are easily analyzed in transforms compared to time. Looking at the time domain, sometimes it’s very messy, I think definitely necessary for a university qualified engineer; I think that technicians don’t use it so much maybe. They might suffice that they just use it superficially as methodology and they don’t bother too much, maybe is behind.</td>
</tr>
<tr>
<td>I16</td>
<td></td>
<td>One difficulty to understand the Laplace transform for the students is that they don’t see the application in the object world, and then they don’t understand what they are doing. They have to use more the abstract thinking to work with changes of spaces (time and frequency domain).</td>
</tr>
<tr>
<td>I15</td>
<td></td>
<td>Many times, the student doesn’t have the enough experience about the “problems of the world” to know how and where he can apply his knowledge and for this reason he cannot see the importance among different topics.</td>
</tr>
</tbody>
</table>

Table 8.3: Comments from teachers

The task for course developers and designers here is to identify, through constructive feedback, the source of these epistemological obstacles and subsequently to free up the
8.3 Relation to our model

The model proposed above by us have some similarities the SOLO and revised Bloom’s taxonomy. However our model focus on the skills and knowledge required by an engineer as expressed by teachers in engineering education and by our analysis. This means that our focus is slightly different from the intentions behind the development of SOLO and Bloom’s taxonomy. We are not elaborating these issues in detail here, but in brief it has bearing on the old, but for a long time overlooked, distinction between episteme and techné as different form of knowledge. Also for example the revised Bloom pre-supposes a linear model for problem solving which requires the problems to be “welldefinied”. However according to Middleton (2002) most problems are “illdefinied”.

In Figure 8.7 we show the different levels of the Techné Pyramid. The level of design in our model is absent or only briefly touched on in the other taxonomies – the level create in the extended Bloom’s taxonomy is not the same as our design and elements in the definition of create is included in our analysis level. In our view the model is nested and within the “pyramid” other “pyramids” could exist (concepts within concepts).

In our analysis level.

Niss (1989) says that the principal aim of education in mathematics is to help the students to become competent and independent subjects. And not victims of the relation between mathematics and society.

For engineering context, the engineer has to solve a problem situation and this situation is concrete and orientated to the reality of the society (having impact, solving necessities, etc.) i.e. Mathematics is a representation of physical things and is a way to understand the natural world, talking about mathematics in it is a way and a reality but focusing in it solves problems in other area, mathematics became a tool.

It is illustrated in this way:

1. The engineering student has to solve a problem situation in a specific context and in it concepts and general elements are involved that we consider as the fundament and corresponding to the first block in the pyramid.

2. Then the next part is when those elements are interacting with each other producing a complex concept and this block become more narrow than the previous because just particular parts of the elements are involved and the abstract thinking are required to develop analysis.

“To acquire a threshold concept a student must integrate it with their personal thinking about their direct and indirect experience of the world. A student who only writes about ‘what economists’ think’ and distances themselves from this has not acquired a threshold concept.”

(Davies and Mangan, 2005)

explains the process of abstraction, generalization and symbolization that are acting in the formation of the mathematical concepts of the following way:

The point is an action or a “system of actions” that can be materials, imagined or symbolic; these actions always are concrete, their elements are a certain objects
Figure 8.7: Levels in Techné Pyramid
(material or ideal). The aim, the meaning and these actions are directed by the attention that a person puts on some relations and connections among the elements of these actions. These relations prove that there is a certain regularity when the actions are repeated (it is to say, invariants of the actions).

3. Finally when the process of analysis solves the problem and it is produces something new the design part is represented as the third part.

4. We present some comments, that are related with our work, about the more prejudicial aspects of the application of the “modern” way to teach mathematics, that Núñez and Font (1995) explain:

<table>
<thead>
<tr>
<th>Nuñez and Font</th>
<th>Comment</th>
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<tbody>
<tr>
<td>a) Exaggerated deductism: mathematics was presented as some knowledges deductively organized. This situation limit to the student stopping his imagination and actions having a finished product.</td>
<td>the process represented in Techné pyramid, promotes the competence and the creativity of the student to go beyond the solution of the problem producing something new.</td>
</tr>
<tr>
<td>b) formalization of definitions. This generate some problems:</td>
<td></td>
</tr>
<tr>
<td>b) 1 To present to students an excess of symbolism.</td>
<td>According with the teachers answers (Study4), some of them point out the necessity in the program of studies to relate theory with practice.</td>
</tr>
<tr>
<td>b) 2 To make them to manipulate mechanically those symbols, without understanding what they are doing (premature formalism).</td>
<td>In Study 1 and 3 the results from students confirm this affirmation in the sense that students are able to give right answers, without understand, in some cases, the meaning of those results.</td>
</tr>
<tr>
<td>b) 3 To forget that to understand a mathematical concept, are necessary reference situations that give sense , at the same time that those situations let to discover the relations with other concepts.</td>
<td>As (I4) commented and confirm this sentence.</td>
</tr>
<tr>
<td>c) Excess of generalization and missing of abstraction process: the concepts where presented in general way.</td>
<td>As the analysis level of Techné Pyramid was explained, it is necessary to narrow the —when we have specific situation for to solve, excluding those elements that are not indispensables and linking those that help us to understand the more complex.</td>
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<tr>
<th>Nuñez and Font</th>
<th>Comment</th>
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<tbody>
<tr>
<td>d) The mathematics for mathematic:where presented mathematics focused in their self and very far of other sciences. The didactic text offered few no mathematical situations and it didn’t let students to know the application of mathematics in the reality, producing questions in students like “and this, for what is usable?”</td>
<td>The importance of the steps in Techné pyramid help us to follow marked objectives, where the end of it is the design. And this design is an application of the reality.</td>
</tr>
</tbody>
</table>

**Table 8.4: Comments on explanations by Nuñez and Font**
Chapter 9

Final Conclusions

There are many factors that can make it difficult for the students in the process of learning, and all of them are worth to study in detail. In our research we identified the topic of the Laplace transform as a difficulty when solving electric circuits. Describing and analyzing that difficulty we propose a model, as a way, to help to understand the complexity of the process of learning in engineering education.

A difficulty (see section 2.4.3) for the engineering students appeared when they did not realize the need to distinguish between alternating and direct current. All students, except one, tried to solve the direct current problem by applying either the Laplace transform model or differential equations. They did so without realizing that the models did not apply to the problem at hand. This is in accordance with both the teaching experience and the results of Studies 2 and 3, concerning how the students interpret concepts and mathematical models used when solving electric circuits. From this we confirm that the engineering students develop systems of thought where they remember certain aspects, elements or conditions of the problems. According to the conclusions of Study 1 (see section 2.5). By recognizing the elements or conditions of the problem they apply the mathematical methods or theorems to solve it. They do not, however, consider the whole context of the problem and, in some cases, they do not understand the physical phenomena that occurs. In the end we question if the students are able to relate the mathematical models with the electrical effect, because they could not explain it in depth (Study 3).

From the research presented earlier in this thesis regarding students’ and teachers’ views about, and understanding of, the Laplace transform and electric circuits the following conclusions are drawn:

9.1 Modeling as the link between the abstract world and the real event

According to the comments from the students in Questionnaire 2 in Study 1 (see section 2.4.3) we found a wide spread of attitudes among the engineering students. It is worrying that the interest of most students is focused only on getting correct answers and passing exams. From personal teaching experience correct, or right, answers does not always mean that the students understand the meaning of the calculations that they make. This was indicated in the interviews with the teachers in Study 4, and also
in the results from Study 3. At the same time we appreciate that the students, when solving problems, sometimes has to handle two separate worlds. On one side they have, and has to consider, the material aspect, on the other they have the abstract aspect. The students has to interact with both these worlds, which is possible through the model. In the case of engineering education the modeling consists of the interaction between the mathematics- and the physics aspects. The students has to be able to, not only join, but also connect and apply them to the actual technology. We propose the model in Figure 9.1 showing the link between the abstract and the material world.

In Figure 9.1 we represent the model as the intersection between the the abstract aspect and the material aspect; where the abstract notion gets a physical meaning. Also we present the interaction between the mathematical, physical and technological aspects as being the modeling in the engineering education context. (As was explained in more detail in section 8.2.)
9.1.1 The Laplace transform as an example

In the interviews with the teachers in Study 4 (section 6.2.7), they affirm that the Laplace transform is more used as a mathematical model. (Teacher A): “From the physical aspect the Laplace transform is a model, and all physical systems has to be represented as a model”, and (Teacher G): “The Laplace transform can be interpreted in the physical world”. (Teacher B): “when we talk with students about zeros, poles, etc. Then we do it in Laplace domain; when we talk about frequency (to increase or reduce frequency) then we do it in Laplace domain (not in time domain) and it simplifies and lets us relate to other tools, as oscilloscopes. For this case, the Matlab program is a good tool of simulation, but it is also important to learn to use it”. The teachers support the views of the students, that the Laplace transform is a complex concept that, sometimes, is used as a model to link abstraction with real event. We can illustrate the “Abstract-Material model” using the Laplace transform with an electric circuit problem (represented in the Figure 9.2):

Where the Laplace transform can be understood as a tool used in, for example, engineering to solve real situations. We describe the process of how the Laplace transform is used in 3 stages, as is illustrated in Figure 9.3:

Where the Laplace transform is used as a tool to solve a real situation interacting with abstraction:

Stage 1 : To represent a physical reality by an electrical model in the time domain. (Material aspect)

Stage 2 : To transform the electrical model in the time domain to a model in the Laplace domain making the calculation easier and obtaining a solution in the Laplace domain. (Abstract aspect)

Stage 3 : To transform the solution in the Laplace domain back to a solution in the original time domain world for the electrical model. (Material aspect)

The incompatibility between theory and practice can generate a deficiency in the development of the learning, this is also expressed in the student comments (section 2.4.3). Nevertheless, this aspect (theory-lab) has been studied, developed and explained more in detail in the model of Carstensen and Bernhard (2004) where they show the different links among the elements involved in understanding the Laplace transform.

One of the aims in engineering education is to make future engineers understand the utility of knowledge when applying it to solve problems. For example, considering
9. Final Conclusions

Figure 9.3: Process of the use of the Laplace transform

the utility of circuit analysis as a tool for the best comprehension of the electronic phenomena and using electronics to put the ideas of analysis and synthesis into practice. In this way the content of the topics in the curriculum must focus on the characteristics of analysis and design by studying the time-dependent phenomena and frequency of the systems applied to the electrical circuits. And, finally, to obtain tools of syntheses, which help to realize the design of circuits. As in many disciplines, the study of the physical electronic systems is done by the formulation and analysis of a mathematical model. In our case, the model is the circuit that is presented in graphical form, as interconnected ideal elements, that can be represented by mathematical models. The analytical solution of the model makes it possible to predict the behavior of the system, and compare these to the results from experiments. In this sense the model is the way to link the abstract and material aspects.

9.2 The Techné-pyramid

In Engineering Education it is necessary to distinguish between the terms *analyzing* and *calculating* since in some situations they are used interchangeably to refer to each other. We claim that it is very important to make this distinction, as we consider the calculations in the first level in the Techné-pyramid, and the analyzing in the second level. Thus, if the students would not be able to combine calculation and analysis they would not be able to design, in the sense of the third level of the Techné-pyramid.
9.2.1 The Circuit Analysis as an example

To illustrate the Techné-pyramid levels we describe the process to analyze electric circuits. Figure 9.4 represent the different topics involved for the analysis of electric circuits: previous knowledges required, parallel topics that are learned, and forward applications. In Figure 9.5 we present the same topic in the context of the Techné Pyramid.

Circuit analysis is between the domains of basic knowledge and applied knowledge. For the analysis notions and ideas of the content that belongs to the basic knowledge is needed, and the understanding of circuit analysis will be determinant for the specific applied knowledges. The engineering students are required to pass the subjects that is concerned with the basic knowledge before starting with the analysis of electric circuits. The following subjects are required:

- Circuit Theory is a compulsory subject that treats the foundations, laws and theorems of the analysis of electric circuits. It contains the introduction to the methods of analysis of linear circuits, principally resistive circuits, and its applications. It establishes the base of knowledge that will be developed and penetrates into the matter of Analysis.

- Algebra and Differential Equations approaches the concepts of complex numbers, differential equations and the Laplace transform. It is important in order to have a good mathematical foundation before starting with circuit analysis.

- Calculus concerns obtaining good levels of confidence and skill in the use of the
Final Conclusions

concepts and technologies of differential and integral calculation and of diverse variables.

When engineering students solve electric circuits problems they are confronting new situations, where it is not enough with only a conceptual comprehension of the problem. In order to solve the problems they have to reach the second level (Analysis level) of the Techné-pyramid, where they are able to link and combine previous and new knowledge, and manage the kind of complex concepts inherent in the problem. This is something that demands a greater effort from the student than they sometimes realize themselves. We say that the students are in a process of “intellectual growth”. In this process it is not sufficient that the students only learn mathematical formulas, and how to apply them. What was previously an achievement for the students to learn, is now a necessary background. When solving electric circuits problems the students face new situations, where they have the options to solve the problems by applying different methods or theories. The student has to choose the most suitable, or convenient, way to solve the problems using all the previously acquired knowledge. Their previous knowledge becomes a tool for the students when the choose, develop, simplify and solve the problems. The problems they are solving are intended to train the students for the kind of professional-life problems they will have to confront later, which also shows the importance of what kind problems the students are solving, and what results they get.

To improve the benefits of learning and to improve the sequence of the content, it is very important to combine simultaneously the analysis of electric circuits with the subjects indicated due to the conceptual existing parallelism.

Concretely:

• To complement the concepts of static and dynamic models of the basic semiconductors devices, as well as to study and to characterize basic circuits and functional subcircuits, which compose the integrated analog circuits raised in the subject of Basic Electronics that allows to the student to have a more wide vision of the electronic applications for those who are prepared for analysis of Circuits.

• To define and to use advanced mathematical tools and software of simulation in the subject of Vectorial Analysis and Fourier. It guarantees the operations and mathematical domains that apply to the matter Circuit Analysis.

The matter of Circuit Analysis must be sufficiently basic to lay the foundations of the methodology of analysis and design of networks and for extension to systems in general, but sufficiently applied to approach the practice and synthesis of the raised topics.

• The deepening in the electronics and the instruments of measure that treat each other in the subjects of analog electronics, Industrial Instrumentation and it associate Laboratories.

• The electrical machines and particularly the study of those in steady-state with sinusoidal voltages and currents.

• The system technologies of control that are approached in the subjects of Theory of Control, Automatic Regulation, Industrial Automation and Laboratory and the System technology of Control.
Figure 9.5: Circuit Analysis in the Techné Pyramid
9.2.2 The Laplace transform as an example

The learning of the Laplace transform is a field that includes part of mathematics, abstract and generalizing science, its laws and forms of characteristic thought. The logical thought and the deductive thought has a specific importance when learning the Laplace transform. Because of this, learning the Laplace transform is a difficult task that contains, in itself, the difficulties it is built on. Figure 9.6 shows the Laplace transform in the context of the Techné Pyramid.

Our primary question is: Why is the Laplace transform taught?

This question can have a number of different answers, but one answer we get from the results of our research is that the Laplace transform is taught for its contribution to the intellectual development of the engineering students. This development is not synonymous to quantitative knowledge or scientific information. The intellectual development has a deeper meaning of developing the skills and capacities and appropriating and using this knowledge to be able to apply it to solve problems. That ability cannot be built by itself, but has to be built on the foundation of the required previous knowledge.

Considering the Laplace transform in the context of the Techné-pyramid, in the first level the Laplace transform is used as substitution of formulas to solve differential equations. At this level the Laplace transform does not require a lot of effort from the students. The teachers interviewed in Study 4 (section 6.2.2) also confirm this, when they say that the Laplace transform is not a difficult topic to learn.

In the second level, when the students has to solve problems that require them to connect different topics in order to solve problems in a specific area, a superficial knowledge of the Laplace transform is not enough. In this level the Laplace transform is interacting with other knowledge in the abstract thinking. The student develops links between “islands of knowledge”, and the previously understood concepts help to understand the complex concepts.

The Laplace transform has applications in different areas, such as automatic control, circuit theory and economics. In the third level of the pyramid the Laplace transform is used as a tool to develop and produce something new in the areas of its application. (See section 6.2.3)

9.3 Cultural observation

As a general observation we noticed from the answers in the questionnaires in Study 3 that the students from Sweden were more inclined to explain and give more arguments than Mexican or Catalans students. For this reason they had a stronger influence in the answers. This difference is cultural but is important to point out that it was not easy to get the students from Mexico and Catalonia to explain the mathematical models they used, because in the engineering context they normally do not justify their procedures and results.
To produce something new
Analysis, abstract thinking.
relation and/or interaction between different elements.
Previous knowledge, complexity and new experiences interacting to solve a problem.
Groups of knowledges, experiences, skills, etc., not necessary linked, either managed in deeper way but are the base and support for other elements.

Figure 9.6: The Laplace Transform in the Techné Pyramid Context
9.4 The Models

With the two models proposed and developed in this thesis called: “Abstract-Material Model” and “Techné-Pyramid” it is possible to understand some difficulties in the process of learning in high levels, especially in engineering, where students has manage threshold concepts to solve specific situations.

“One of my students told me: No teacher!, the Laplace transform is like a black box that nobody understand and it only serves to torture students”

This kind of expression is quite common to hear in among students when they have to use certain threshold concepts. However, as was explained before, the Laplace transform is one topic among others that requires certain elements as a background to understand it, but to use it for some specific application, it has to be used in interaction with other topics. And we observe that the topic is of greater importance in engineering education, than it is usually given in the actual study programs. We notice a strong limitation of the use of this topic (as others) in the sense that it is often taught only superficially. For this reason it is no surprise to notice that the students do not find it relevant. Summarizing:

1. The Laplace transform is used as a way to solve problems facilitating solutions linking the abstract and material aspect.

2. The Laplace transform is a topic used to analyze solving problems and require previous knowledges to understand it.
The aim to learn the Laplace transform topic is that the engineering student acquire elements that lead it to improving his interpretation of the world, to acquire major intellectual maturity and develop his own vision linking mathematics and physics as part of his environment developing knowledge, skills, capacities and mental attitudes to achieve the creative thinking and a critical attitude that allows him to generate his own strategies of thought solving problems like a part of his integral formation. This will increase his capacity for the cognitive independence and it will improve his social performance and in the world of the work.
9.5 Mediation through tools/artifacts

A central point in this thesis is a discussion about the Laplace transform as a tool for use in engineering study and practice. Therefore a short overview of some theories about tool/artifact use will give an important theoretical background to this thesis.

In the socio-cultural theory of learning (also called cultural-historical school) developed by Vygotsky (e. g. Vygotsky, 1978; Cole, 1996; Wertsch, 1998; Kozulin, 1998; Kozulin et al., 2003) and his co-workers and students the concept of “tool” and “mediation” is central. Much of human interaction with the material and social world is not direct but mediated through the use of mediating tools that could be expressed as: Human ⇐⇒ Tools ⇐⇒ World. The central thesis in this school is that the structure and development of human psychological processes are shaped by the interaction with tools. These are historically developed and could be of different types such as “psychological tools”, “material tools” and also language counts as a tool. Using tools make more powerful and functional ways of acting possible and would enhance human development.

A reason for the powerfulness of using tools is that tools have knowledge of previous generations and the knowledge of its developers built in. In the socio-cultural theory the mediating tools are seen as simultaneously ideal (conceptual) and material. For this reason Cole (1996) proposes that the term “artifact” should be used for both psychological and material tools ("psychological" tools are also “material” since our brain is material). However it should be noted that in the literature both usages are common: That the term artifact refers only to ”material” tools and to wider usage proposed by Cole. In this thesis I will use the term tool when discussing modeling and
the Laplace-transform.

From the perspective that artifacts fuse ideality and materiality Cole (1996) argue:

“This view also also asserts the primal unity of the material and the symbolic in human cognition /.../ [and] provides a way of dealing with long-standing debate /.../: Should culture be located external to the individual, as the products of prior human activity, or should it be located internally, as a pool of knowledge and beliefs? /.../ The concepts of artifacts as products of human history that are simultaneously ideal and material offers a way out of this debate.”

In chapter 7 we had expressed similar ideas when we introduce modeling as a link between the “material” and “abstract” world.

The idea of tool mediation is not original to the socio-cultural school but is also central in some schools within philosophy of technology extending the works of either Dewey Hickman (see for example 1990) or Heidegger (1977).

Dewey insisted that theory is a kind of practice. According to Hickman (1990) Dewey “render a unique contention that tools or instruments cut across traditional boundary lines such as those between the psychical and the physical, the inner and the outer, and the real and the ideal.” For Dewey “inquiry was /.../ a productive skill whose artifact is knowing” (cf. the discussion about techné as a special kind of knowledge earlier in this thesis). Hickman further argued “Unlike most philosophers of technology, Dewey held the view that technological instruments include immaterial objects such as ideas, theories, numbers, and the objects of logic /.../. His instrumentalist account of inquiry rejected both realism and idealism on the grounds that neither position was capable of developing and adequate understanding of the function played in knowing by tools and media of all sorts.” Further which is important for this thesis is that “Dewey specifically associates tool use in its most important sense with knowing ...” and for him both screwdrivers and mathematical concepts were tools.

In the book Thinking through Technology Mitcham (1994) also discusses mediation and extends the work of Ihde (1979, 1983) and introduces the model Human Instrument World. This work is further extended in the recent book What Things Do by Verbeek (2005). According to him “[T]he concept of mediation helps to show that technologies actively shape the character of human-world relations. Human contact with reality is always mediated, and technologies offer one possible form of mediation. On the other hand, it means that any particular mediation can only arise within specific contexts of use and interpretation.”

It is beyond the scope of this thesis to go into a detailed discussion about the issues raised in the books mentioned above. However some questions raised and discussed by Verbeek (2005) are central for us: “What role do technological artifacts play in the manner in which human beings interpret reality?” and what are the “acts of artifacts”.

In our case we have studied the role of the Laplace transform and how the use of different tools (in general sense) can be understood in light of the abstract-material model and the Techne Pyramid. We think that these models connects well to the theories for mediated learning and will contribute to the improvement of engineering education.
9. Final Conclusions